Third International Symposium on Computational Mechanics (ISCM III)
Second Symposium on Computational Structural Engineering (CSE II)
December 5-7, 2011, Taipei, TAIWAN

Automatic Structural Analysis with Boundary Face Method and Adaptive Cross Approximation

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Within a successful CAE driven product development, there are two fundmental issues: a seamless interaction between the CAD and CAE software tools and a fast solver for very large scale computation. The first issue is associated with the discretization of any complexly shaped domain into elements for a mesh-based method, such as the Finite Element Method (FEM) or Boundary Element Method (BEM). In the best case modifications in the CAD model can be transferred to the CAE model and vice versa. The second issue is inevitable for a daily design process, as a complete simulation chain for practical problems usually involves very large scale computations and can be very long.

To render a standard approach that may ensure a seamless interaction between CAD and CAE, several kinds of boundary type meshless method have been proposed, such as the Boundary Node Method (BNM)[1] and the Hybrid Boundary Node Method (HdBNM). The BNM combines the moving least-squares (MLS) approximation with the Boundary Integral Equations (BIE) in order to retain both the meshless attribute of the former and the dimensionality advantage of the latter. Unfortunately, the BNM still uses the standard elements for boundary integration and approximation of the geometry, thus loses the advantages of the meshless methods. This work presents a new implementation of the boundary node method (BNM). In this implementation, here called the boundary face method (BFM)[2], the boundary integration is performed on boundary faces, which are represented in parametric form exactly as the boundary representation (Brep) data structure in solid modelling. The integrand quantities, such as the coordinates of Gauss integration points, Jacobian and out normal are calculated directly from the faces rather than from elements. The resulted method requires a nodal data structure on the bounding surface of a body only, and hence has potential to make direct use of a body's parametric representation, which is available in most of CAD packages. In order to deal with thin structures, a mixed variable interpolation scheme of 1-D MLS and Lagrange Polynomial for long and narrow faces. An adaptive integration scheme for nearly singular integrals has also been developed.

Due to the non-local kernels of the integral operator in the BIE, the coefficient matrix is fully populated. This leads to that both the memory requirement and CPU time for solving the system equation are of $O(N^2)$ complexity, where N is the number of unknowns. Several methods that dramatically reduce memory and computational cost have been developed in the last two decades. Among these methods the most popular ones are the Fast Multiple Method (FMM) [3], the Hierarchical Matrix[4] and the Adaptive Cross Approximation (ACA) [5]. The FMM reduces numerical complexity to $O(N)$. However, the implementation of the FMM depends on a priori knowledge of the kernel function, which is to be expanded by spherical harmonic series. The Hierarchical Matrix combined with ACA, taking advantage of the rank-deficient nature of the coupling matrix blocks representing well-separated clusters interactions, is a purely algebraic algorithm, namely the computational speed-up is achieved through linear algebra manipulations of the matrix, e.g. QR decomposition, SVD, LU decomposition, etc. Due to its algebraic nature, ACA can be modular and very easily integrated into various existing BEM codes. In this work, we will adopt the ACA to accelerate the BFM computation.

So far, in most implementations of the fast BEM, constant elements are used. As constant elements are discontinuous, they are independent to each other. The simple connectivity between constant elements makes it easy to computation cluster to cluster interactions in FMM or other acceleration techniques. Moreover, as analytical integrals on a constant element are available, there is no need to evaluate the boundary integrals numerically. This largely simplifies the boundary integration computation and reduces memory requirement. However, constant elements are not efficient to approximate the geometry of a surface with small radius of curvature. More importantly, the results for stresses at boundary points obtained with constant elements are often inaccurate, as the derivatives of shape functions of constant elements are zero. The BFM is a general framework in which any kinds of shape function can be used. In the current implementation, the shape function from MLS has been employed, thus the numerical integration is inevitable as there is not even an explicit expression for the MLS shape functions. In order to integrate the numerical integration into the ACA, we have elaborately developed a general algorithm to connect near-field and far-field integrations.

The combination between the BFM and ACA results in an efficient algorithm not only in terms of computational cost but also in terms of human-labor cost, as the work for preparing the computational model is greatly simplified. A number of application examples, including those from real world product design, are presented.

References

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